The Construction of Quantitative Reciprocal Reasoning

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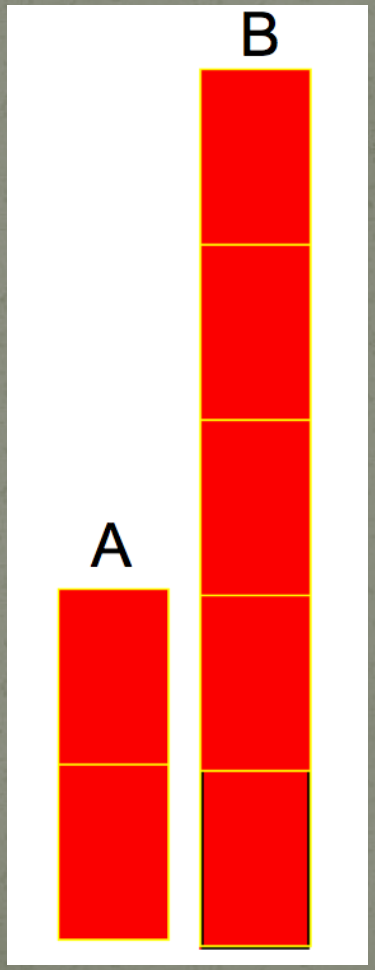
Purpose

- Show how students who have interiorized three levels of units constructed quantitative reciprocal reasoning.
  - I refer to these students as students who have constructed the third multiplicative concept, or MC3 students.
- Comment on a learning trajectory for quantitative reciprocal reasoning for MC3 students.

Quantitative Reciprocal Reasoning

- If A is 2/5 of B, then…
- B is 5/2 of A because 1/5 of B is ½ of A, so 5/5 of B has to be the same as 5/2 of A.
- *NOT:* B is 5/2 of A because you use the reciprocal.

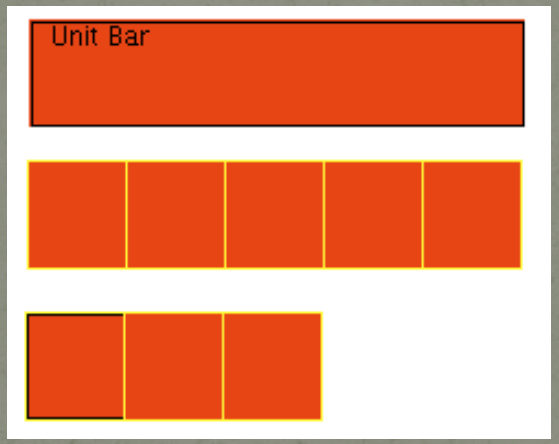


Two Studies

- Study 1: Interview study with 18 middle and high school students, 6 of which were MC3 students.
  - 3 constructed quantitative reciprocal reasoning;
  - Interviewer learned about how to support construction of reciprocal reasoning in interaction.
- Study 2: Design experiments with 18 middle school students (so far), 6 of which were/are MC3 students.
  - All 6 constructed quantitative reciprocal reasoning.

A Quantitative Approach

- Fractions as quantities



- Unknowns as potential measurements of quantities

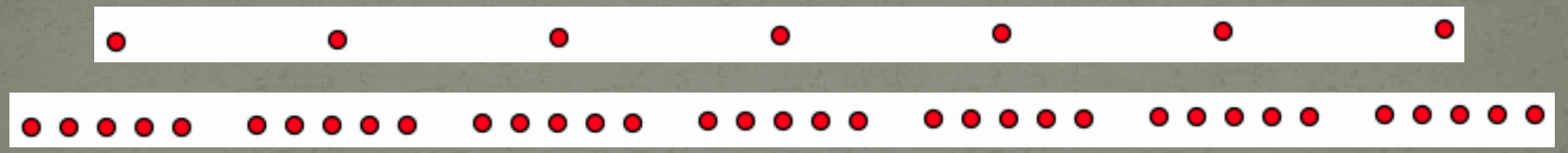
Tools for Modeling Students' Thinking and Learning

- Operations and schemes
  - Partitioning, iterating, disembedding
  - Iterative fraction scheme: fractional numbers (Steffe & Olive, 2010)

- Accommodation
  - Horizontal and vertical learning

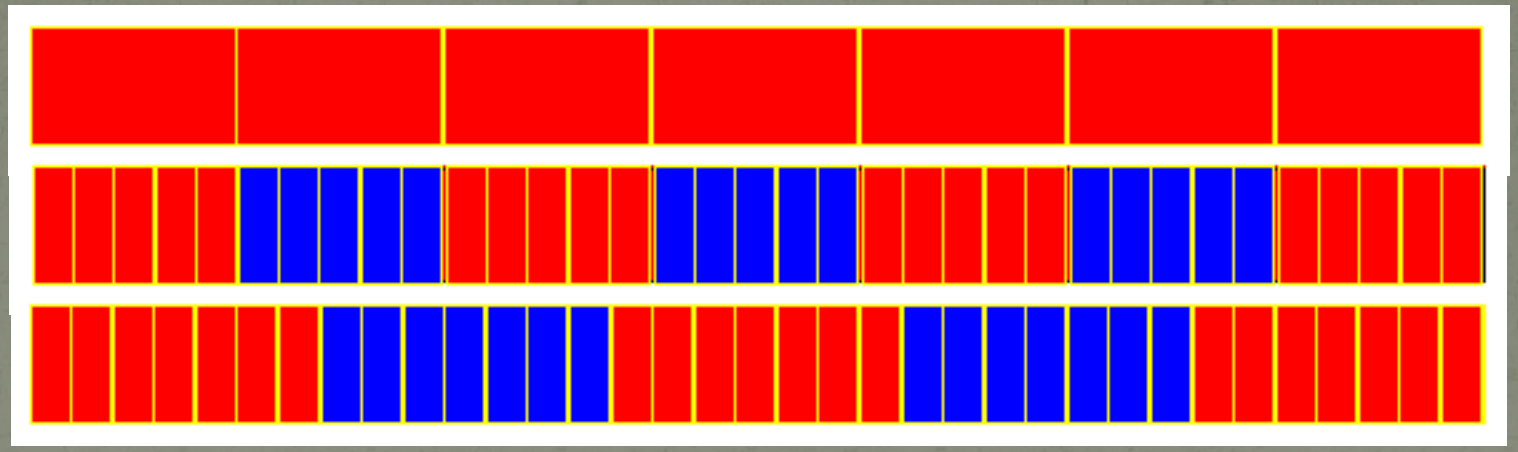
Students' Multiplicative Concepts

- *Concept:* interiorized result of a scheme.
- *Composite unit*: a unit of units.
- *Units coordination*: distribute the units of one composite unit across the elements of another composite unit.



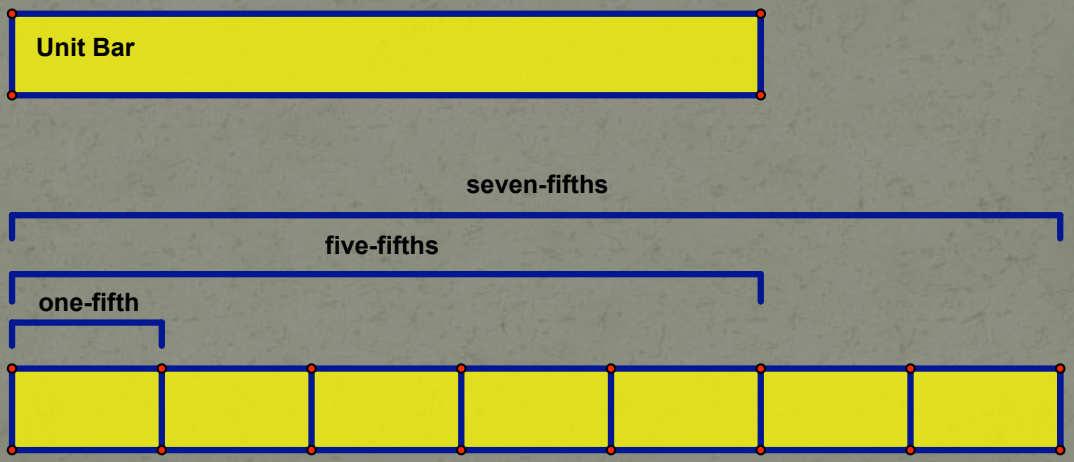
MC3 Students

- Can take three levels of units as given and flexibly switch between three-levels-of-units structures.
- Maintain views of 35 as a unit of 7 units each containing 5.



- Can construct an iterative fraction scheme.

Iterative Fraction Scheme



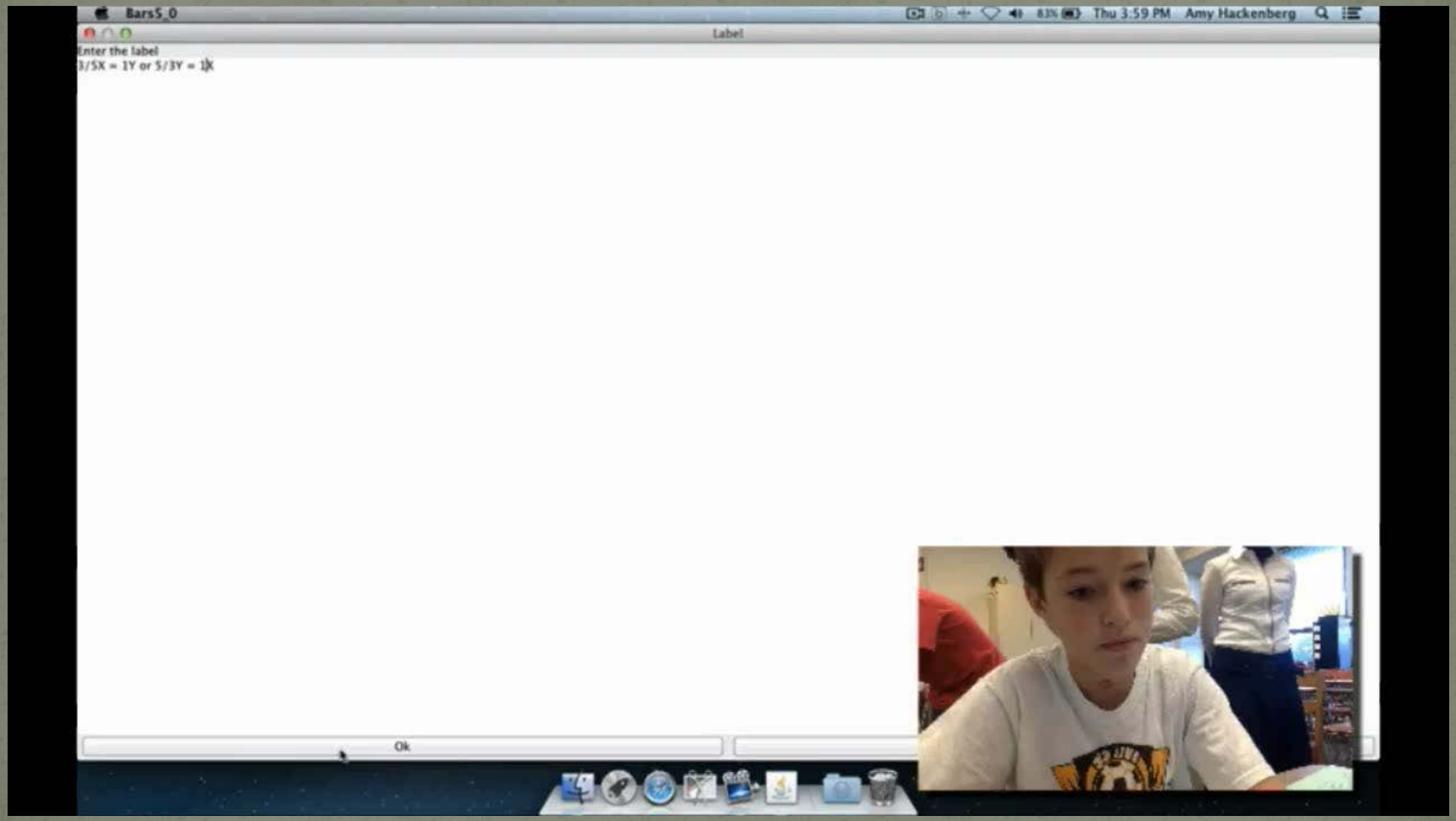
Possible Outcome of Making Models: Learning Trajectory

- Based on work of Clements & Sarama, 2009; Simon, 1995; Steffe, 2004:
  - Initial model of students' current schemes, operations, and concepts
  - Account of observable changes in those ways of operating
  - Account of mathematical interactions involved in the changes

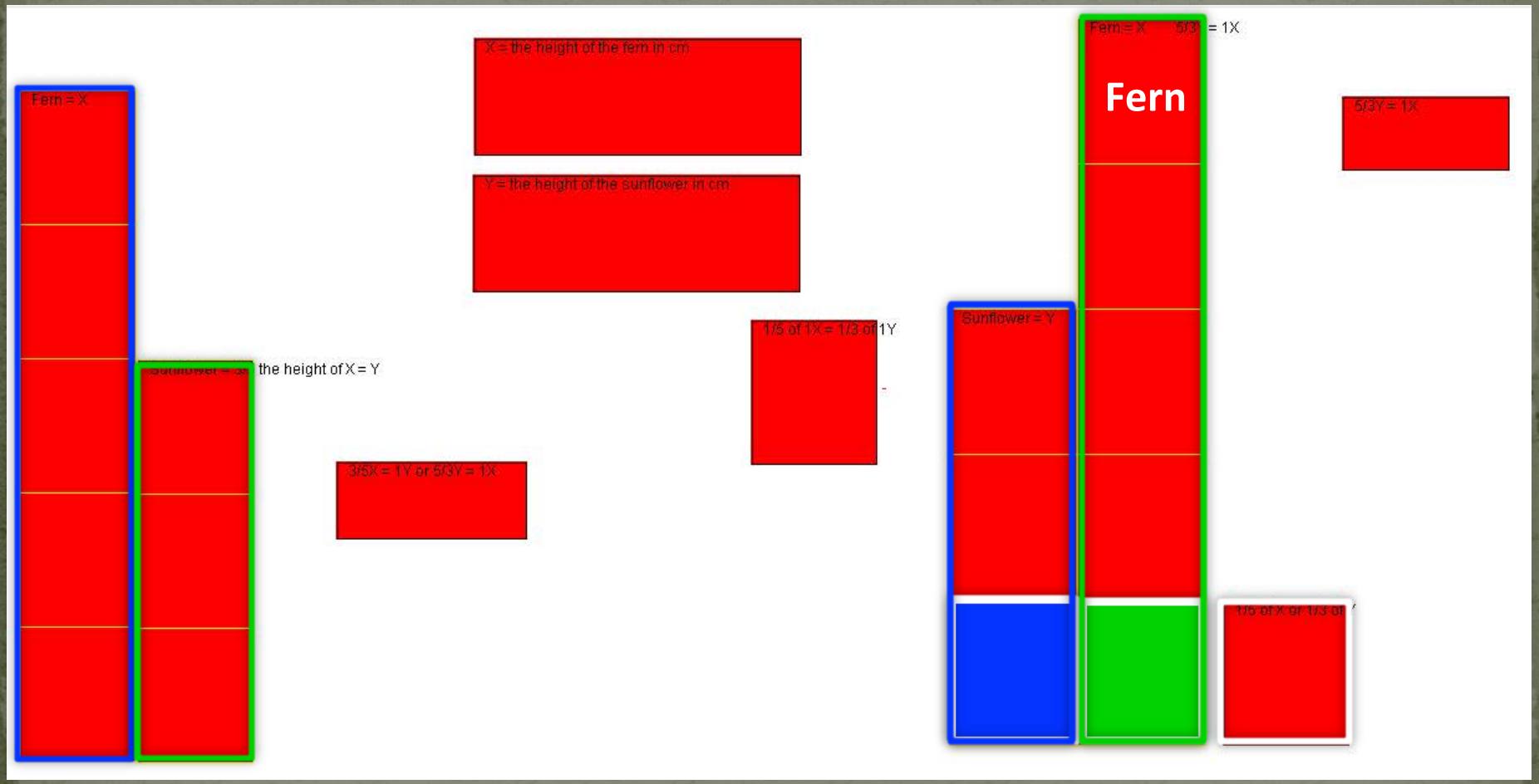
Fern-Sunflower Problem

- ***Fern-Sunflower Problem*:** A fern and sunflower are growing in the garden, each of unknown height. The height of the sunflower is 3/5 the height of the fern.
  - Draw a picture of this situation and describe what your picture represents.
  - Write an equation for this situation that relates the two heights. Explain your equation in terms of your picture.
  - Can you write another, different equation that relates the two heights? Explain this equation in terms of your picture.

Martin (7th) and Gabriel (8th)



Martin's work on JavaBars



switch between referent unit

Pivotal moments in construction of QRR

- Martin, 7th grade: "... but instead ... the sunflower is the main piece and that's three parts and this [fern's height] is five and this is one that's one whole bar plus two fifths so, and that would be five-thirds..."
- Andrea, 7th grade: "It's looking at it in the height of the sunflower, not the height of the fern."
- Hector, 10th grade: "... if [the sunflower's height] can be considered a whole, it can be considered a whole but that's just to itself, so [the fern's height] is more than the whole. So it's more than one hundred percent, or an improper fraction."

Toward a Learning Trajectory for QRR

- The construction of QRR involves an accommodation in one's iterative fraction scheme, at least.
- What features of students' interactions are important?
  - Focus on quantitative basis/meaning for writing equations.
  - Moving back and forth between algebraic notation and representation of the quantitative situation in drawings.
  - Highlighting moves the teacher-researcher sees as productive, e.g., determining that one part of a height can be named in two ways, depending on referent unit.

IDR²eAM Project Members

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- Erol Uzan

IDR²eAM stands for Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School.